The influence of compressibility on the stresses of elastic porous solids—semimicroscopic investigations

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Abstract

In this paper the influence of the compressibility of the real material of the constituents of a porous medium on the stresses will be discussed for a simplified model of liquid-saturated porous solids. The basis of the model is the mixture theory restricted by the volume fraction condition (theory of porous media). In comparison with the mixture theory, one additional constitutive relation for the so-called real part of the deformation of the solid phase will be formulated to close the system of equations for compressible binary porous media within the framework of the theory of porous media. The real deformation can be described by a second order tensor which results from the multiplicative decomposition of the deformation gradients of solid and liquid constituents. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

In contrast to a mixture, a porous medium, i.e. a porous solid skeleton filled with an arbitrary number of liquids, presents an internal structure, namely the pore structure. In general, the pore structure shows a complicated geometry so that it is almost impossible to separate the constituents and to use the classical continuum mechanics to describe the behavior of the separated constituents in consideration of all boundary and initial conditions (microscopic description of a porous medium). For this reason, a macroscopic theory has been developed and is still under study concerning the description of the thermodynamic behavior of porous media. The basis of this theory is the mixture theory. In order to identify the partial bodies, the so-called concept of volume fractions has been introduced. This concept is understood as the determination of the fractions of a body occupied by a constituent φ^α (the superscript α denotes individual constituents, for example a solid phase $\alpha = S$, a liquid phase $\alpha = L$) with the local ratio of the corresponding partial volume (volume of the constituent φ^α) in relation to the total volume (volume of the control space, which

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is shaped by the solid skeleton). Readers interested in the foundation of porous media theories (continuum theories of mixtures in consideration of the concept of volume fractions) are referred to, e.g. Drumheller (1978), Bowen (1980, 1982), de Boer and Ehlers (1986), Ehlers (1989), de Boer (1991, 1996a) and de Boer et al. (1991).

However, the extension of the mixture theory by the concept of volume fractions makes no sense if there is no coupling between the volume fractions and the field quantities of the mixture theory. Therefore, the consideration of the concept of volume fractions is connected with the introduction of so-called real densities of the constituents (Fillunger, 1936). The real density of φ^α is a quantity which transfers one important material behavior from the microscale to the macroscale, namely whether the real material of a constituent is compressible or incompressible.

It is well known that there is another crucial point in the development of a consistent theory for porous media. The introduction of the concept of volume fractions results in 'smeared' continua. This, along with the saturation condition (i.e. that the sum of the volume fractions of all constituents is equal to one), is faced with the problem that for a saturated porous media consisting of κ compressible constituents, $\kappa-1$ field equations are missing. The quantity $\kappa-1$ arises from the number of the volume fractions of the constituents together with the additional field equation, namely the saturation condition. In order to close the system of field equations for compressible porous media, Drumheller (1978), Bowen (1982) and Ehlers (1993) have formulated additional evolution equations for $\kappa-1$ volume fractions. Another possibility has been indicated by Nunziato and Walsh (1980) and by Passmann et al. (1984). They have formulated additional balance equations for the so called 'equilibrated forces', which are in the opinion of the authors not physically well founded. With respect to the closure problem, equally it is possible to introduce additional constitutive equation (de Boer, 1992). The problem regarding the missing field equations does not occur in the mixture theory. Within the framework of the mixture theory it is not possible to decide whether a constituent is compressible or incompressible.

The goal of the present work is to discuss the influence of the compressibility of the real materials of the constituents on the stresses for a liquid-saturated elastic porous solid. With respect to the description of incompressibility and compressibility of the real material of the constituents a purely kinematic concept will be used. This concept is based on a multiplicative decomposition of the deformation gradient of the constituent φ^α, which is a helpful tool for transferring the deformation behavior of a constituent at the microscale to the macroscale (Bluhm and de Boer, 1997). In order to close the system of field equations, a constitutive relation for the real deformation of the solid phase will be formulated, for which it will be assumed that the rotation of the real deformation part of the constituent solid has no influence on the stress state of the porous solid. The results obtained from the presented model regarding the constitutive relations for the stresses will be compared with experimental observations and other well-known models.

2. Concept of volume fractions and kinematics

The concept of volume fractions in addition to the mixture theory has shown itself to be an efficient tool for investigating saturated as well as empty porous solids. With the assumption that the pores are statistically distributed over the porous solid, this concept effects the distribution of the mass of the constituents over the control space of the porous solid. Thus, with the concept of

volume fractions the single constituents are 'smeared' over the control space of the solid phase and occupy the whole volume of the control space simultaneously. The 'smeared' substitute continua (partial bodies) for the solid and liquid phases of a binary porous medium show the same properties as a mixture body, so that the mixture theory is the most adequate basis for a thermodynamic treatment of porous media at the macroscale.

In contrast to mixtures, with respect to the description of porous media, it is of essential importance that the partial bodies of a porous medium be 'identifiable' at any time during a thermodynamic process. The identification will be done with the help of the concept of volume fractions.

The actual volume v of a liquid filled porous solid body B_S is composed additively by the actual partial volumina v^α of the two constituents φ^S (solid) and φ^L (liquid):

$$v = v^S + v^L = \int_{B_S} (dv^S + dv^L) = \int_{B_S} dv, \tag{2.1}$$

where dv^α and dv are the actual volume elements of the single constituent and the total body, respectively. With the volume fractions (the concept of volume fractions is supported by micromechanics, de Boer and Didwania, 1997)

$$n^\alpha = n^\alpha(\mathbf{x}, t) \tag{2.2}$$

at each point $\mathbf{x}$ of the control space B_S and at each time t, provided with the property

$$dv^\alpha = n^\alpha \, dv, \tag{2.3}$$

the total volume v can be represented as

$$v = \int_{B_S} (n^S + n^L) \, dv. \tag{2.4}$$

A comparison between eqn (2.1) and eqn (2.4) yields

$$n^S + n^L = 1. \tag{2.5}$$

Equation (2.5) is the so-called saturation condition, which plays an important role in the theory of porous media because this condition (2.5) restricts the motion of the individual constitutions.

As has already been mentioned, the concept of volume fractions has the effect of distributing the mass of the individual constituents over the control space of the porous solid. Taking the mass of the constituent φ^α with regard to the actual placement,

$$M^\alpha = \int_{B_S} \rho^{\alpha R} \, dv^\alpha = \int_{B_S} n^\alpha \rho^{\alpha R} \, dv = \int_{B_S} \rho^\alpha \, dv, \tag{2.6}$$

one obtains the following expression for the density ρ^α of the 'smeared' substitute continua of φ^α (partial density):

$$\rho^\alpha = n^\alpha \rho^{\alpha R}, \tag{2.7}$$

where $\rho^{\alpha R}$ is the so-called real density of φ^α. The real density is a quantity which transfers one

important material behavior from the microscale to the macroscale, namely whether the real material of a constituent is compressible or incompressible, i.e. whether the real density of the constituent is constant or not.

Within the framework of the general porous media theory, a liquid-saturated porous medium will be treated as an immiscible mixture of all constituents φ^α with particles X_α, where at any time t each spatial point of the current placement of the solid phase (φ^S) is simultaneously occupied by particles X_L of the constituent liquid (φ^L). These particles proceed from different reference positions $\mathbf{X}_L$ at time $t = t_0$. Thus, each constituent is assigned its own independent motion function:

$$\mathbf{x} = \mathscr{X}_\alpha(\mathbf{X}_\alpha, t), \tag{2.8}$$

where $\mathbf{x} \in B_S$. In general, the reference positions $\mathbf{X}_L$ of the liquid particles do not need to be elements of the solid body B_{0S} of the reference placement ($\mathbf{X}_L \notin B_{0S}$). Only for deformation processes, in which the liquid phase flows out of the control space of the solid phase, are the reference positions $\mathbf{X}_L$ elements of B_{0S} ($\mathbf{X}_L \in B_{0S}$).

Using the Lagrange description of motion, eqn (2.8), the velocity and the acceleration field of the constituent φ^α are defined as material time derivatives of the motion function (2.8):

$$\mathbf{x}'_\alpha = \frac{\partial \mathscr{X}_\alpha(\mathbf{X}_\alpha, t)}{\partial \mathbf{t}}, \quad \mathbf{x}''_\alpha = \frac{\partial^2 \mathscr{X}_\alpha(\mathbf{X}_\alpha, t)}{\partial t^2}. \tag{2.9}$$

The function $\mathscr{X}_\alpha$ is postulated to be unique and uniquely invertible at any time t. The existence of a function inverse to eqn (2.8) leads to the Eulerian description of motion, viz:

$$\mathbf{X}_\alpha = \mathscr{X}_\alpha^{-1}(\mathbf{x}, t). \tag{2.10}$$

By using eqn (2.10), one obtains an alternative representation of the velocity and acceleration fields:

$$\mathbf{v}_\alpha = \mathbf{x}'_\alpha = \mathbf{x}'_\alpha[\mathscr{X}_\alpha^{-1}(\mathbf{x}, t), t] = \mathbf{x}'_\alpha(\mathbf{x}, t),$$

$$\mathbf{a}_\alpha = \mathbf{x}''_\alpha = \mathbf{x}''_\alpha[\mathscr{X}_\alpha^{-1}(\mathbf{x}, t), t] = \mathbf{x}''_\alpha(\mathbf{x}, t). \tag{2.11}$$

For vector and scalar fields depending on $\mathbf{x}$ and t, the material time derivatives are defined as

$$(\ldots)'_\alpha = \frac{\partial(\ldots)}{\partial t} + [\mathrm{grad}(\ldots)]\mathbf{x}'_\alpha, \quad (\ldots)'_\alpha = \frac{\partial(\ldots)}{\partial t} + \mathrm{grad}(\ldots) \cdot \mathbf{x}'_\alpha. \tag{2.12}$$

A mathematically sufficient condition for the existence of eqn (2.10) is given if the Jacobian

$$J_\alpha = \det \mathbf{F}_\alpha \tag{2.13}$$

differs from zero. From the physical point of view J_α must be positive. In eqn (2.13) $\mathbf{F}_\alpha$ is the deformation gradient of the constituent φ^α, which is defined as

$$\mathbf{F}_\alpha = \frac{\partial \mathscr{X}_\alpha(\mathbf{X}_\alpha, t)}{\partial \mathbf{X}_\alpha} = \mathrm{Grad}_\alpha \mathscr{X}_\alpha. \tag{2.14}$$

The material velocity gradient $(\mathbf{F}_\alpha)'_\alpha$ and spatial velocity gradient $\mathbf{L}_\alpha$ read as follows:

$$(\mathbf{F}_\alpha)'_\alpha = \frac{\partial \mathbf{x}'_\alpha}{\partial \mathbf{X}_\alpha} = \mathrm{Grad}_\alpha \, \mathbf{x}'_\alpha, \quad \mathbf{L}_\alpha = (\mathrm{Grad}_\alpha \, \mathbf{x}'_\alpha)\mathbf{F}_\alpha^{-1} = \mathrm{grad} \, \mathbf{x}'_\alpha, \tag{2.15}$$

where the inverse of the deformation gradient $\mathbf{F}_\alpha$ is given by

$$\mathbf{F}_\alpha^{-1} = \frac{\partial \mathcal{X}_\alpha^{-1}(\mathbf{x}, t)}{\partial \mathbf{x}} = \frac{\partial \mathbf{X}_\alpha}{\partial \mathbf{x}} = \mathrm{grad} \, \mathbf{X}_\alpha. \tag{2.16}$$

The spatial velocity gradient $\mathbf{L}_\alpha$ can be additively decomposed into a symmetric and a skew-symmetric part denoted by $\mathbf{D}_\alpha$ and $\mathbf{W}_\alpha$, respectively:

$$\mathbf{L}_\alpha = \mathbf{D}_\alpha + \mathbf{W}_\alpha, \quad \mathbf{D}_\alpha = \frac{1}{2}(\mathbf{L}_\alpha + \mathbf{L}_\alpha^\mathrm{T}), \quad \mathbf{W}_\alpha = \frac{1}{2}(\mathbf{L}_\alpha - \mathbf{L}_\alpha^\mathrm{T}). \tag{2.17}$$

The reader, who is interested in a review of the kinematics of porous media is referred to, e.g. de Boer and Ehlers (1986) and Ehlers (1989).

3. Field equations

The field equations for porous media consist of the balance equations of the constituents taken from the mixture theory and the saturation condition. Excluding mass exchanges between the solid and liquid phases and neglecting additional supply terms of moment of momentum, the local field equations for an isothermal process for a binary porous medium are given by the local statements of the balance equations of mass,

$$(\rho^\mathrm{S})'_\mathrm{S} + \rho^\mathrm{S} \, \mathrm{div} \, \mathbf{x}'_\mathrm{S} = 0, \quad (\rho^\mathrm{L})'_\mathrm{L} + \rho^\mathrm{L} \, \mathrm{div} \, \mathbf{x}'_\mathrm{L} = 0, \tag{3.1}$$

the balance equations of momentum,

$$\mathrm{div} \, \mathbf{T}^\mathrm{S} + \rho^\mathrm{S}(\mathbf{b}^\mathrm{S} - \mathbf{x}''_\mathrm{S}) = -\hat{\mathbf{p}}^\mathrm{S}, \quad \mathrm{div} \, \mathbf{T}^\mathrm{L} + \rho^\mathrm{L}(\mathbf{b}^\mathrm{L} - \mathbf{x}''_\mathrm{L}) = -\hat{\mathbf{p}}^\mathrm{L}, \tag{3.2}$$

where

$$\hat{\mathbf{p}}^\mathrm{S} + \hat{\mathbf{p}}^\mathrm{L} = \mathbf{o}, \tag{3.3}$$

and the saturation condition (2.5) together with the relation (2.7),

$$n^\mathrm{S} + n^\mathrm{L} = 1, \quad n^\mathrm{S} = \frac{\rho^\mathrm{S}}{\rho^\mathrm{SR}}, \quad n^\mathrm{L} = \frac{\rho^\mathrm{L}}{\rho^\mathrm{LR}}. \tag{3.4}$$

In the field eqns (3.2) and (3.3) the quantities $\rho^\alpha \mathbf{b}^\alpha$ and $\hat{\mathbf{p}}^\alpha$ denote the external body force and the supply term of momentum of the constituent φ^α, respectively. The tensor $\mathbf{T}^\alpha$ is the partial Cauchy stress tensor of φ^α. The statement (3.3) with respect to the supply terms of momentum is a result of the assumption that the sum of the balance equations over all constituents (balance equations of the mixture) must be formally equivalent to the corresponding conservation laws (balance laws without supply terms) of a one-component material (Truesdell's third 'metaphysical principle', Truesdell, 1984). Furthermore, it has been pointed out that additional supply terms of moment of

momentum will be neglected. Thus, it is not necessary to present the balance equations of moment of momentum, which only yield the statement that the partial stress tensors are symmetric:

$$\mathbf{T}^\alpha = (\mathbf{T}^\alpha)^\mathrm{T}. \tag{3.5}$$

A broad review of the balance equations of the porous media and mixture theory, respectively, is given by de Boer (1996a) and Bluhm (1997).

The usual way to describe microscopic phenomena such as compressibility and incompressibility of the real material is to introduce the real densities $\rho^{\alpha R}$ with the coupling of these densities, due to the saturation condition, in the set of process variables. The possibility to describe microscopic properties by macroscopic quantities is a motivation to try a more sophisticated and elegant procedure for expressing kinematic quantities of the microscale by quantities of the macroscale. The normal way, i.e. the introduction of a motion function on the microscale which serves as the basis for all further kinematic quantities, e.g. strains and velocity of the real material points, as in eqn (2.8), is not possible. Thus, no balance equations exist in the porous media theory for determining this micromechanical quantity. Therefore, it is advisable to transfer the microscopic deformation behavior of the constituents φ^α to the macroscale. For that reason the deformation tensor $\mathbf{F}_{\alpha R}$, which is understood as a part of the deformation gradient $\mathbf{F}_\alpha$, is introduced, and it is assumed to reflect the microscopic deformations of the real material of the constituent φ^α on the macroscale. In general, the tensor $\mathbf{F}_{\alpha R}$ is not integrable on the macroscale, i.e. the microscopic deformations are represented by incompatible deformations on the macroscale. Furthermore, $\mathbf{F}_{\alpha R}$ is generally different from $\mathbf{F}_\alpha$ ($\mathbf{F}_\alpha \neq \mathbf{F}_{\alpha R}$), and it is necessary to choose a second tensor $\mathbf{F}_{\alpha N}$ in order to transfer this inequality into an equation. The part $\mathbf{F}_{\alpha N}$ of the deformation gradient $\mathbf{F}_\alpha$, as well as $\mathbf{F}_{\alpha R}$, is in general not integrable. On the contrary, the deformation tensor $\mathbf{F}_\alpha$ is integrable; thus, the deformation gradient on the macroscale $\mathbf{F}_\alpha$ must be multiplicatively decomposed into the parts $\mathbf{F}_{\alpha R}$ and $\mathbf{F}_{\alpha N}$:

$$\mathbf{F}_\alpha = \mathbf{F}_{\alpha N}\mathbf{F}_{\alpha R}. \tag{3.6}$$

The tensor $\mathbf{F}_{\alpha N}$ is supposed to describe the remaining part of the deformation of the control space, namely, the change of the pores in size and shape.

The aforementioned range and the usefulness of the multiplicative decomposition has already been shown by Bluhm and de Boer (1997). From the mathematical point of view, $\mathbf{F}_{\alpha N}$ and $\mathbf{F}_{\alpha R}$ have to be understood as local mappings of tangent (vector) spaces in each material point of the partial body, i.e. the multiplicative decomposition (3.6) is generally connected with the suggestion of an incompatible intermediate placement which reflects the deformations of the real material. It is worth mentioning that the multiplicative decomposition is compatible with an additive decomposition of strains and strain rates as in the finite elasto-plasticity theory; (Lee, 1969; Haupt, 1985). For example, the Lagrange strain tensor of the solid phase assigned to the porosity and the pore structure, with respect to the reference placement of φ^S, can be expressed as the difference between the Lagrange strain tensors of the partial solid body and the real solid phase (Bluhm, 1997),

$$\mathbf{E}_{SN} = \mathbf{E}_S - \mathbf{E}_{SR} = \frac{1}{2}(\mathbf{F}_S^\mathrm{T}\mathbf{F}_S - \mathbf{F}_{SR}^\mathrm{T}\mathbf{F}_{SR}) = \frac{1}{2}(\mathbf{C}_S - \mathbf{C}_{SR}), \tag{3.7}$$

where $\mathbf{C}_S$ and $\mathbf{C}_{SR}$ are the right Cauchy–Green tensors of the partial and real solid phase.

In the following, the influence of the decomposition (3.6) will be discussed in view of the dependence of $\mathbf{F}_\alpha$ on the volume fraction, the real density of φ^α and the determinants of the two deformation parts of $\mathbf{F}_\alpha$.

Considering eqn (3.6), the determinant J_α of the deformation gradient $\mathbf{F}_\alpha$ can be written as

$$J_\alpha = \det \mathbf{F}_\alpha = \det \mathbf{F}_{\alpha\mathrm{N}} \det \mathbf{F}_{\alpha\mathrm{R}} = J_{\alpha\mathrm{N}} J_{\alpha\mathrm{R}}. \tag{3.8}$$

By using eqns (2.7) and (3.8), the local statement (3.1) of the balance equation of mass of the constituent φ^α can be replaced by

$$\rho^{\alpha\mathrm{R}}[(n^\alpha)'_\alpha + n^\alpha J_{\alpha\mathrm{N}}^{-1}(J_{\alpha\mathrm{N}})'_\alpha] + n^\alpha[(\rho^{\alpha\mathrm{R}})'_\alpha + \rho^{\alpha\mathrm{R}} J_{\alpha\mathrm{R}}^{-1}(J_{\alpha\mathrm{R}})'_\alpha] = 0. \tag{3.9}$$

As eqn (3.9) is valid for arbitrary $\rho^{\alpha\mathrm{R}}$ and n^α, the terms in the brackets must be equal to zero in order to fulfil the relation (3.9). By using the Lagrange description of motion, the time integration of the individual expressions yields

$$J_{\alpha\mathrm{N}} = \frac{n^\alpha_{0\alpha}}{n^\alpha}, \quad J_{\alpha\mathrm{R}} = \frac{\rho^{\alpha\mathrm{R}}_{0\alpha}}{\rho^{\alpha\mathrm{R}}}, \tag{3.10}$$

where $n^\alpha_{0\alpha}$ and $\rho^{\alpha\mathrm{R}}_{0\alpha}$ denote the volume fraction and the real density of φ^α at the position $\mathbf{X}_\alpha$ at time $t = t_0$. The insertion of eqn (3.10) into eqn (3.8) yields

$$J_\alpha = \frac{n^\alpha_{0\alpha}}{n^\alpha} \frac{\rho^{\alpha\mathrm{R}}_{0\alpha}}{\rho^{\alpha\mathrm{R}}} = \frac{\rho^\alpha_{0\alpha}}{\rho^\alpha}. \tag{3.11}$$

The latter relation ($J_\alpha = \rho^\alpha_{0\alpha}/\rho^\alpha$) is also the result of the time integration of the local balance equations of mass, eqn (3.1), regarding φ^α.

With the help of the decompositions (3.6) and (3.8), respectively, e.g. the incompressibility of the solid phase can be formulated by a kinematic condition, namely $J_{\mathrm{SR}} = 1$, as in the classical continuum mechanics for one component solid materials.

Considering the multiplicative decomposition of $\mathbf{F}_\alpha$, the model presented here for a binary porous medium will be described by 24 field equations,

$$\{\text{Number of field equations}\} = 24, \tag{3.12}$$

viz., the balance equations of mass and momentum (8 equations), eqns (3.1) and (3.2), the local statement concerning the supply terms of momentum, eqn (3.3) (3 equations), the saturation condition (3.4)$_\mathrm{a}$ (1 equation), the relations (3.4)$_\mathrm{b,c}$ regarding the volume fractions (2 equations), the additive decomposition (3.7) of the Lagrange strain tensors of the solid phase (6 equations) and the results, eqn (3.10), with respect to the determinants $J_{\alpha\mathrm{N}}$ and $J_{\alpha\mathrm{R}}$ (4 equations).

The decomposition (3.6) of $\mathbf{F}_\alpha$ will not be considered in the set of field equations. Only the influence of eqn (3.6) on the third invariant of the deformation gradient of the solid and liquid phase and the additive decomposition of the strains of the solid phase will be taken into consideration. Thus, the volume-preserving part of the liquid phase and the rotational part of the deformation of the real material of the solid phase, i.e. the rotation of the intermediate placement of φ^S, will not be considered in the present model.

4. Constitutive theory

In the field equations mentioned previously, there exists in total 55 field variables, where 10 quantities are known:

$$\mathscr{K} = \{\mathbf{b}^\alpha, n^\alpha_{0\alpha}, \rho^{\alpha R}_{0\alpha}\} \Rightarrow \mathscr{V}^{10}. \tag{4.1}$$

In order to close the system of equations for the model of a binary porous medium consisting of compressible phases (i.e. the number of field equations must be equal to the number of unknown field quantities) constitutive relations will be formulated for the following set of field quantities:

$$\mathscr{C} = \{\mathbf{T}^\alpha, \hat{\mathbf{p}}^L, \mathbf{C}_{SR}\} \Rightarrow \mathscr{V}^{21}. \tag{4.2}$$

Thus, list of the unknown field quantities reads:

$$\mathscr{U} = \{\mathscr{X}_\alpha, \hat{\mathbf{p}}^S, \rho^\alpha, n^\alpha, \rho^{\alpha R}, J_{\alpha N}, J_{LR}, \mathbf{E}_{SN}\} \Rightarrow \mathscr{V}^{24}, \tag{4.3}$$

where $J_{SR} = \sqrt{\det \mathbf{C}_{SR}}$ has been considered. The symbol $\mathscr{V}^{(\cdots)}$ denotes the dimension of the vector space of the list of the corresponding field quantities. A comparison of eqns (3.12) and (4.3) shows that the number of field equations is equal to the number of unknown field quantities.

Concerning the thermodynamic restrictions relevant for the formulation of constitutive relations for the field quantities summarized in eqn (4.2), one has to account for the entropy inequality of the mixture. For an isothermal process, the entropy inequality for a binary mixture is given by (de Boer, 1996a):

$$-\rho^S(\psi^S)'_S - \rho^L(\psi^L)'_L + \mathbf{T}^S \cdot \mathbf{D}_S + \mathbf{T}^L \cdot \mathbf{D}_L - \hat{\mathbf{p}}^L \cdot (\mathbf{x}'_L - \mathbf{x}'_S) \geqslant 0. \tag{4.4}$$

The quantity ψ^α denotes the free Helmholtz energy function of the constituent φ^α.

As mentioned before, the saturation condition $(3.4)_a$ is understood as a constraint, i.e. as a restriction on the motion of the solid and the liquid phase, and must be accounted for in the entropy inequality. The influence of the saturation constraint on the inequality (4.4) will be taken into account by using the material time derivative of the saturation condition $(3.4)_a$ following the motion of the solid or the liquid phase, together with the concept of Lagrange multipliers. Here, the material time derivative of the saturation condition following the motion of the solid phase will be used:

$$(n^S)'_S + (n^L)'_S = 0. \tag{4.5}$$

By using eqns $(2.12)_b$, (3.8) and $(3.10)_a$ and the material time derivatives

$$(J_S)'_S = J_S(\mathbf{D}_S \cdot \mathbf{I}), \quad (J_L)'_L = J_L(\mathbf{D}_L \cdot \mathbf{I}),$$
$$(J_{SR})'_S = J_{SR}(\hat{\mathbf{D}}_{SR} \cdot \mathbf{I}), \quad (J_{LR})'_L = J_{LR}(\bar{\mathbf{D}}_{LR} \cdot \mathbf{I}), \tag{4.6}$$

the derivatives $(n^S)'_S$ and $(n^L)'_S$ read as follows:

$$(n^S)'_S = -n^S(\mathbf{D}_S \cdot \mathbf{I}) + n^S(\hat{\mathbf{D}}_{SR} \cdot \mathbf{I}),$$
$$(n^L)'_S = -n^L(\mathbf{D}_L \cdot \mathbf{I}) + n^L(\bar{\mathbf{D}}_{LR} \cdot \mathbf{I}) - \operatorname{grad} n^L \cdot (\mathbf{x}'_L - \mathbf{x}'_S). \tag{4.7}$$

The multiplication of eqn (4.5) by the Lagrange multiplier λ and (-1), and the insertion of eqn (4.7) into eqn (4.5), yield the constraint

$$n^S \lambda (\mathbf{D}_S \cdot \mathbf{I}) - n^S \lambda (\hat{\mathbf{D}}_{SR} \cdot \mathbf{I}) + n^L \lambda (\mathbf{D}_L \cdot \mathbf{I}) - n^L \lambda (\bar{\mathbf{D}}_{LR} \cdot \mathbf{I}) + \lambda \operatorname{grad} n^L \cdot (\mathbf{x}'_L - \mathbf{x}'_S) = 0. \tag{4.8}$$

The symbols ($\hat{}$) and ($\bar{}$) denote the quantities which refer to the local intermediate placement of the solid and liquid phase. The Lagrange multiplier λ is understood as the reaction force of the saturation constraint. Now, eqn (4.8) will be added to the entropy inequality (4.4), leading to:

$$-\rho^S (\psi^S)'_S - \rho^L (\psi^L)'_L - \{\hat{\mathbf{p}}^L - \lambda \operatorname{grad} n^L\} \cdot (\mathbf{x}'_L - \mathbf{x}'_S) + \mathbf{D}_S \cdot \{\mathbf{T}^S + n^S \lambda \mathbf{I}\} - \hat{\mathbf{D}}_{SR} \cdot \{n^S \lambda \mathbf{I}\}$$
$$+ \mathbf{D}_L \cdot \{\mathbf{T}^L + n^L \lambda \mathbf{I}\} - \bar{\mathbf{D}}_{LR} \cdot \{n^L \lambda \mathbf{I}\} \geqslant 0. \tag{4.9}$$

One recognizes that the Lagrange multiplier λ is connected with, among others, the material time derivative of J_{SR} and $\mathbf{C}_{SR}$, respectively, along the trajectory of φ^S. As $\mathbf{C}_{SR}$ is a constitutive field quantity in the model, one has to consider the constitutive relation for $\mathbf{C}_{SR}$ in the entropy inequality. With the constitutive assumption that $\mathbf{C}_{SR}$ is a function of the process variable $\mathbf{C}_S$,

$$\mathbf{C}_{SR} = \mathbf{C}_{SR}(\mathbf{C}_S), \tag{4.10}$$

the material time derivative $(\mathbf{C}_{SR})'_S$ can be written as

$$(\mathbf{C}_{SR})'_S = 2\mathbf{F}_{SR}^T \hat{\mathbf{D}}_{SR} \mathbf{F}_{SR} = \frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S} (\mathbf{C}_S)'_S = 2 \frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S} (\mathbf{F}_S^T \mathbf{D}_S \mathbf{F}_S). \tag{4.11}$$

Using eqn (4.11), the trace of $\hat{\mathbf{D}}_{SR}$ in eqn (4.9) can be replaced by

$$\hat{\mathbf{D}}_{SR} \cdot \mathbf{I} = \frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S} (\mathbf{F}_S^T \mathbf{D}_S \mathbf{F}_S) \cdot \mathbf{C}_{SR}^{-1} = \mathbf{F}_S \left[\left(\frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S} \right)^T \mathbf{C}_{SR}^{-1} \right] \mathbf{F}_S^T \cdot \mathbf{D}_S, \tag{4.12}$$

and the entropy inequality of the binary porous medium reads:

$$-\rho^S (\psi^S)'_S - \rho^L (\psi^L)'_L + \{\hat{\mathbf{p}}_E^L\} \cdot (\mathbf{x}'_L - \mathbf{x}'_S) + \mathbf{D}_S \cdot \{\mathbf{T}_E^S\} + \mathbf{D}_L \cdot \{\mathbf{T}_E^L\} - \bar{\mathbf{D}}_{LR} \cdot \{n^L \lambda \mathbf{I}\} \geqslant 0. \tag{4.13}$$

The entropy inequality (4.13) represents an important relation for formulating restrictions for the so-called 'effective' partial stresses $\mathbf{T}_E^S$ and $\mathbf{T}_E^L$ and the 'effective' interaction force $\hat{\mathbf{p}}_E^L$. These 'effective' quantities are defined as

$$\mathbf{T}_E^S = \mathbf{T}^S + n^S \lambda \left\{ \mathbf{I} - \mathbf{F}_S \left[\left(\frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S} \right)^T \mathbf{C}_{SR}^{-1} \right] \mathbf{F}_S^T \right\},$$

$$\mathbf{T}_E^L = \mathbf{T}^L + n^L \lambda \mathbf{I}, \quad \hat{\mathbf{p}}_E^L = -\hat{\mathbf{p}}^L + \lambda \operatorname{grad} n^L. \tag{4.14}$$

The so-called 'effective' quantities are determined by the motion of the solid and/or liquid phase. Furthermore, the definition of the 'effective' quantities in eqn (4.14) is associated with the change of the list of the constitutive field quantities (4.2). Considering eqns (4.10), (4.13), (4.14) and the preceding remarks, the list of the 'effective' constitutive quantities reads:

$$\mathscr{C}_E = \{\mathbf{T}_E^\alpha, \hat{\mathbf{p}}_E^L\} \Rightarrow \mathscr{V}^{15}. \tag{4.15}$$

It will be further assumed that the free Helmholtz energy functions ψ^S and ψ^L and the extra supply term of momentum $\hat{\mathbf{p}}_E^L$ are determined by

$$\psi^S = \psi^S(\mathbf{C}_S), \quad \psi^L = \psi^L(J_{LR}), \quad \hat{\mathbf{p}}_E^L = \hat{\mathbf{p}}_E^L(\mathbf{x}_L' - \mathbf{x}_S'), \tag{4.16}$$

compare with de Boer (1997). Thus, the inequality (4.13) can be transferred into:

$$\mathbf{D}_S \cdot \left\{\mathbf{T}_E^S - 2\rho^S \mathbf{F}_S \frac{\partial \psi^S}{\partial \mathbf{C}_S} \mathbf{F}_S^T\right\} + \mathbf{D}_L \cdot \{\mathbf{T}_E^L\}$$

$$-\bar{\mathbf{D}}_{LR} \cdot \left\{n^L \lambda \mathbf{I} + n^L \rho^{LR} J_{LR} \frac{\partial \psi^L}{\partial J_{LR}} \mathbf{I}\right\} + (\mathbf{x}_L' - \mathbf{x}_S') \cdot \{\hat{\mathbf{p}}_E^L\} \geqslant 0, \tag{4.17}$$

where eqns (3.4)$_c$, (4.6)$_d$ and the expression for $(\mathbf{C}_S)_S'$ in eqn (4.11) have been used.

The evaluation of the inequality (4.17) for the free available quantities $\mathbf{D}_S$, $\mathbf{D}_L$ and $\bar{\mathbf{D}}_{LR}$ yields the following constitutive relations for $\mathbf{T}_E^S$, $\mathbf{T}_E^L$ and λ:

$$\mathbf{T}_E^S = 2\rho^S \mathbf{F}_S \frac{\partial \psi^S}{\partial \mathbf{C}_S} \mathbf{F}_S^T, \quad \mathbf{T}_E^L = \mathbf{0}, \quad \lambda = p = -\rho^{LR} J_{LR} \frac{\partial \psi^L}{\partial J_{LR}}. \tag{4.18}$$

Regarding the interpretation of $\lambda = p$ as the pore pressure of φ^L, the reader is referred to de Boer (1996a).

The dissipation mechanism of the inequality (4.17) reads:

$$\mathscr{D} = \hat{\mathbf{p}}_E^L \cdot (\mathbf{x}_L' - \mathbf{x}_S') \geqslant 0. \tag{4.19}$$

The insertion of the ansatz

$$\hat{\mathbf{p}}_E^L = \gamma(\mathbf{x}_L' - \mathbf{x}_S') \tag{4.20}$$

into eqn (4.19) yields the condition that the dissipation mechanism is fulfilled if the parameter γ is zero or is positive:

$$\gamma \geqslant 0. \tag{4.21}$$

The partial stress tensors and the supply term of momentum of φ^L for the compressible binary porous medium as defined in eqn (4.14) now read:

$$\mathbf{T}^S = -n^S p \left\{\mathbf{I} - \mathbf{F}_S \left[\left(\frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S}\right)^T \mathbf{C}_{SR}^{-1}\right] \mathbf{F}_S^T\right\} + \mathbf{T}_E^S,$$

$$\mathbf{T}^L = -n^L p \mathbf{I}, \quad \hat{\mathbf{p}}^L = -\hat{\mathbf{p}}_E^L + p \operatorname{grad} n^L, \tag{4.22}$$

where $\mathbf{T}_E^S$, $p = \lambda$ and $\hat{\mathbf{p}}_E^L$ have been identified in eqns (4.18)$_{a,c}$ and (4.20).

By using (3.10)$_b$, the derivative of the free Helmholtz energy function of φ^L, with respect to J_{LR} in (4.18)$_c$, can be replaced by

$$\frac{\partial \psi^L}{\partial J_{LR}} = \frac{\partial \psi^L}{\partial \rho^{LR}} \frac{\partial \rho^{LR}}{\partial J_{LR}} = -\frac{\rho^{LR}}{J_{LR}} \frac{\partial \psi^L}{\partial \rho^{LR}}, \tag{4.23}$$

where the free Helmholtz energy ψ^L is a function of ρ^{LR}, i.e. $\psi^L = \psi^L(\rho^{LR})$. Therefore, the pore pressure p of the liquid phase in eqn (4.18)$_c$ can be written as

$$p = (\rho^{LR})^2 \frac{\partial \psi^L}{\partial \rho^{LR}}. \tag{4.24}$$

Concerning the results for a hybrid model consisting of a compressible or an incompressible solid phase and an incompressible liquid phase as well as for an incompressible model, the reader is referred to de Boer (1996b).

In the remaining part of this chapter, the influence of the real part of the deformations of the solid phase together with the pressure of the liquid will be discussed for a simplified model, and a comparison will be made between the model for a binary porous medium and the results of other models and test observations.

The sum of eqns (4.22)$_a$ and (4.22)$_b$ yields the local statement

$$\mathbf{T}^{S+L} = \mathbf{T}^S + \mathbf{T}^L = -p \left\{ \mathbf{I} - n^S \mathbf{F}_S \left[\left(\frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S} \right)^T \mathbf{C}_{SR}^{-1} \right] \mathbf{F}_S^T \right\} + \mathbf{T}_E^S \tag{4.25}$$

for the total stress state of the binary porous medium. Equation (4.25) clearly reveals that the total stress state of a liquid-saturated porous solid is composed of two parts; one part is connected with the pore-liquid pressure and the other part is governed by the motion of the partial solid phase. Only if the influence of the deformations of the real solid phase can be neglected, i.e. $\mathbf{C}_{SR} = \mathbf{I}$, can the relation (4.25) correspond to von Terzaghi's statement of the effective stress 'principle' (von Terzaghi, 1936):

$$\mathbf{T}^{S+L} = -p\mathbf{I} + \mathbf{T}_E^S. \tag{4.26}$$

By decomposing $\mathbf{C}_S$ into a spherical part $J_S^{2/3}\mathbf{I}$ and a volume-preserving part $\check{\mathbf{C}}_S$,

$$\mathbf{C}_S = J_S^{2/3} \check{\mathbf{C}}_S, \quad \det \check{\mathbf{C}}_S = 1, \tag{4.27}$$

the function $\mathbf{C}_{SR} = \mathbf{C}_{SR}(\mathbf{C}_S)$ can be transferred into

$$\mathbf{C}_{SR} = \mathbf{C}_{SR}(J_S, \check{\mathbf{C}}_S). \tag{4.28}$$

Thus, the partial derivative of $\mathbf{C}_{SR}$ with respect to $\mathbf{C}_S$ can be replaced by

$$\frac{\partial \mathbf{C}_{SR}}{\partial \mathbf{C}_S} = \frac{1}{2} J_S \left(\frac{\partial \mathbf{C}_{SR}}{\partial J_S} \otimes \mathbf{C}_S^{-1} \right) + J_S^{-2/3} \left\{ \frac{\partial \mathbf{C}_{SR}}{\partial \check{\mathbf{C}}_S} - \frac{1}{3} \left(\frac{\partial \mathbf{C}_{SR}}{\partial \check{\mathbf{C}}_S} \mathbf{C}_S \otimes \mathbf{C}_S^{-1} \right) \right\}, \tag{4.29}$$

and in consideration of

$$\mathbf{C}_{SR} = J_{SR}^{2/3} \check{\mathbf{C}}_{SR}, \quad \det \check{\mathbf{C}}_{SR} = 1, \tag{4.30}$$

the expression $\mathbf{F}_S[(\partial \mathbf{C}_{SR}/\partial \mathbf{C}_S)^T \mathbf{C}_{SR}^{-1}]\mathbf{F}_S^T$ in eqn (4.25) can be written as

$$\mathbf{F}_{\mathrm{S}}\left[\left(\frac{\partial \mathbf{C}_{\mathrm{SR}}}{\partial \mathbf{C}_{\mathrm{S}}}\right)^{\mathrm{T}} \mathbf{C}_{\mathrm{SR}}^{-1}\right]\mathbf{F}_{\mathrm{S}}^{\mathrm{T}} = J_{\mathrm{SR}}^{-1}J_{\mathrm{S}}\frac{\partial J_{\mathrm{SR}}}{\partial J_{\mathrm{S}}}\mathbf{I} + \frac{1}{2}J_{\mathrm{S}}\left(\frac{\partial \check{\mathbf{C}}_{\mathrm{SR}}}{\partial J_{\mathrm{S}}}\cdot\check{\mathbf{C}}_{\mathrm{SR}}^{-1}\right)\mathbf{I} + 2J_{\mathrm{SR}}^{-1}J_{\mathrm{S}}^{-2/3}\left[\mathbf{F}_{\mathrm{S}}\frac{\partial J_{\mathrm{SR}}}{\partial \check{\mathbf{C}}_{\mathrm{S}}}\mathbf{F}_{\mathrm{S}}^{\mathrm{T}}\right]^{\mathrm{D}}$$

$$+ J_{\mathrm{S}}^{-2/3}\left\{\mathbf{F}_{\mathrm{S}}\left[\left(\frac{\partial \check{\mathbf{C}}_{\mathrm{SR}}}{\partial \check{\mathbf{C}}_{\mathrm{S}}}\right)^{\mathrm{T}}\check{\mathbf{C}}_{\mathrm{SR}}\right]\mathbf{F}_{\mathrm{S}}^{\mathrm{T}}\right\}^{\mathrm{D}}. \quad (4.31)$$

If the influence of the deviatoric deformation part of the real solid material is neglected on the macroscale,

$$\check{\mathbf{C}}_{\mathrm{SR}} = \mathbf{I}, \quad (4.32)$$

then eqn (4.31) can be simplified to:

$$\mathbf{F}_{\mathrm{S}}\left[\left(\frac{\partial \mathbf{C}_{\mathrm{SR}}}{\partial \mathbf{C}_{\mathrm{S}}}\right)^{\mathrm{T}}\mathbf{C}_{\mathrm{SR}}^{-1}\right]\mathbf{F}_{\mathrm{S}}^{\mathrm{T}} = J_{\mathrm{SR}}^{-1}J_{\mathrm{S}}\frac{\partial J_{\mathrm{SR}}}{\partial J_{\mathrm{S}}}\mathbf{I}. \quad (4.33)$$

The insertion of eqn (4.33) into eqn (4.25) yields the following relation for the total stress tensor:

$$\mathbf{T}^{\mathrm{S}+\mathrm{L}} = -p\left(1 - n^{\mathrm{S}}J_{\mathrm{S}}J_{\mathrm{SR}}^{-1}\frac{\partial J_{\mathrm{SR}}}{\partial J_{\mathrm{S}}}\right)\mathbf{I} + \mathbf{T}_{\mathrm{E}}^{\mathrm{S}}. \quad (4.34)$$

By using the constitutive relation

$$J_{\mathrm{SR}} = (J_{\mathrm{S}})^{m}, \quad m = \frac{k^{\mathrm{S}}}{k^{\mathrm{SR}}}, \quad (4.35)$$

where k^{S} and k^{SR} represent the compression moduli of the partial solid body and the real solid material (de Boer, 1996a; Bluhm and de Boer, 1996), the derivation of J_{SR} referring to J_{S} multiplied by $J_{\mathrm{S}}J_{\mathrm{SR}}^{-1}$ in eqn (4.34) reads as

$$J_{\mathrm{S}}J_{\mathrm{SR}}^{-1}\frac{\partial J_{\mathrm{SR}}}{\partial J_{\mathrm{S}}} = \frac{k^{\mathrm{S}}}{k^{\mathrm{SR}}}. \quad (4.36)$$

The insertion of eqn (4.36) into eqn (4.34) yields

$$\mathbf{T}^{\mathrm{S}+\mathrm{L}} = -p\left(1 - n^{\mathrm{S}}\frac{k^{\mathrm{S}}}{k^{\mathrm{SR}}}\right)\mathbf{I} + \mathbf{T}_{\mathrm{E}}^{\mathrm{S}}. \quad (4.37)$$

For a purely hydrostatic stress state, it follows from eqn (4.37) that

$$p^{\mathrm{S}+\mathrm{L}} = -p\left(1 - n^{\mathrm{S}}\frac{k^{\mathrm{S}}}{k^{\mathrm{SR}}}\right) + p_{\mathrm{E}}^{\mathrm{S}}, \quad (4.38)$$

where the notations

$$p^{\mathrm{S}+\mathrm{L}} = \frac{1}{3}(\mathbf{T}^{\mathrm{S}+\mathrm{L}}\cdot\mathbf{I}), \quad p_{\mathrm{E}}^{\mathrm{S}} = \frac{1}{3}(\mathbf{T}_{\mathrm{E}}^{\mathrm{S}}\cdot\mathbf{I}) = \rho^{\mathrm{S}}J_{\mathrm{S}}\frac{\partial \psi^{\mathrm{S}}}{\partial J_{\mathrm{S}}} \quad (4.39)$$

have been used regarding the total partial hydrostatic stress state of the porous body and the

effective hydrostatic stresses of the solid phase. Equation (4.38) represents a version of the effective stress 'principle' originally proposed by Suklje (1969). The validity of this formula for hydrostatic stress states has been recently proved experimentally by Lade and de Boer (1997).

By using the constitutive assumption

$$J_{SR} = \frac{1}{n_{0S}^S k^{SR}} [k^S (J_S - 1) + n_{0S}^S k^{SR}] \tag{4.40}$$

instead of eqn (4.35), and considering eqns (3.8) and (3.10)$_a$, the total hydrostatic stress state reads

$$p^{S+L} = -p \left(1 - \frac{k^S}{k^{SR}} \right) + p_E^S. \tag{4.41}$$

It is worth mentioning that the principle of effective stresses in the form of eqn (4.41) has already been formulated by Biot and Willis (1957) for the theory of consolidation. The relation (4.41) of the concept of effective stresses has been derived by Nur and Byerlee (1971) for wet rocks under high hydrostatic pressure. In order to demonstrate the physical validity of eqn (4.41), Nur and Byerlee (1971) made several simple compression and pore pressure tests on Weber sandstone, for example. It is worth noting that the porosity of the tested Weber sandstone is nearly 6%, i.e. $n^S = 0.94$. Thus, the calculation of the effective hydrostatic pressure of the solid phase by using eqn (4.41) or eqn (4.38) yields nearly the same results, so that, in the opinion of the authors, the experiments of Nur and Byerlee (1971) do not clearly show whether the ratio of the compression moduli is connected with the volume fraction of the solid or not.

The constitutive eqn (4.32) for $\check{\mathbf{C}}_{SR}$ and eqn (4.35) or eqn (4.40) for J_{SR} are only valid for the description of processes with small deformations of the solid phase. Therefore, the effective stress tensor of the constituent φ^S can be determined with the help of a constitutive relation in the form of Hooke's law, regarding the reference placement of φ^S together with the Lagrange strain tensor $\mathbf{E}_S$, see eqn (3.7):

$$\mathbf{T}_E^S = \mathbf{S}_E^S = 2\mu^S \mathbf{E}_S + \lambda^S (\mathbf{E}_S \cdot \mathbf{I}) \mathbf{I}, \quad \mathbf{E}_S = \frac{1}{2} (\mathbf{F}_S^T \mathbf{F}_S - \mathbf{I}). \tag{4.42}$$

In eqn (4.42), the quantities $\mathbf{S}_E^S$, μ^S and λ^S are the effective symmetric Piola–Kirchhoff stress tensor and the so-called Lamé constants of the solid phase. Furthermore, in eqn (4.42) it has been considered that, for small deformations, the effective Cauchy stress tensor is equal to the corresponding symmetric Piola–Kirchhoff stress tensor, i.e. $\mathbf{T}_E^S = \mathbf{S}_E^S$.

Assuming also small deformations for the constituent φ^L, the constitutive equation for the real pore-liquid pressure for a compressible liquid phase in support of Hooke's law concerning φ^S reads:

$$p = k^{LR} \left(1 - \frac{\rho_{0L}^{LR}}{\rho^{LR}} \right) = k^{LR} (1 - J_{LR}) = -k^{LR} e_{LR}, \tag{4.43}$$

where k^{LR} denotes the compression modulus of the real liquid material, and the term $e_{LR} = J_{LR} - 1$ represents the microscopic volume deformation of the real liquid phase at the macroscale (de Boer,

1996a). If the liquid phase is incompressible, the pore pressure is an indeterminate quantity in the model.

5. Conclusions

The above investigations show that in the presented model for a liquid-saturated porous solid, considering the deformations of the real material of the constituents, the effective stresses of the solid phase are influenced by the reduced pore liquid pressure. The reduction factor depends on the real material property and the deformations of the real solid material. Furthermore, it has been shown that, by neglecting the volume-preserving part of the real solid deformations, the results of the constitutive equations for the stresses of the binary model can be transferred to the corresponding constitutive assumptions of the models of Suklje (1969) and Nur and Byerlee (1971). Also, von Terzaghi's statement of the effective stress 'principle' (von Terzaghi, 1936) can be derived (Lade and de Boer, 1997).

The results show that further experimental investigations are needed to clarify the influence of a volume-preserving deformation part of the real solid phase on the principle of effective stresses. From the physical point of view, the aforementioned deformation part must be considered in the constitutive relation for the partial stress tensor of the solid constituent. The reason is that, for an incompressible solid phase, the volume-preserving deformations of the real solid material on the microscale are responsible for the macroscopic deformations. This is the case, for example, for a porous solid filled with an incompressible liquid phase under a hydrostatic stress state with drained surfaces. Consequently the aim of further investigations will be to study the principle of effective stresses for compressible porous media in consideration of all deformations of the real solid material on the microscale.

Acknowledgements

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